Homework 1

Number Theory and Cryptography (201912400327)

Due Date: May 20, 2024

Question 1.

Compute the greatest common divisor $gcd(455, 1235)$ by hand.

Question 2.

Let $a, b, n \in \mathbb{N}$ be natural numbers.

- What is $\gcd(n, 0), \gcd(n, 1), \gcd(n, n), \gcd(n, 2n)$?
- Show that $\gcd(a, a + b) = \gcd(a, b)$.
- Show that $\gcd(\frac{a}{\gcd(a,b)}, \frac{b}{\gcd(a.b)}) = 1$.

Question 3.

- Show that $\gcd(n, n + 1) = 1$ for any $n \in \mathbb{Z}$.
- Show that $\gcd(22n + 7, 33n + 10) = 1$ for any $n \in \mathbb{Z}$.

Question 4.

Show that there are infinitely many primes of the form $4n + 3$.

Question 5.

The Fibonacci numbers F_n are defined recursively via $F_0 = 0,\ F_1 = 1,$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. The first few F_n are given by $0, 1, 1, 2, 3, 5, 8, 13, 21, \cdots$

- Show that $\gcd(F_n, F_{n+1}) = 1$, for all $n \in \mathbb{N}$.
- Prove Honsberger's identity:

$$F_{m+n} = F_m F_{n+1} + F_{m-1} F_n,\ \text{for all } m, n \in \mathbb{N}.$$

- Show that $m \mid n$ implies that $F_m \mid F_n$.